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PRECISION METHODS OF NONDESTRUCTIVE CONTROL OF THERMOPHYSICAL PROPERTIES

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UDC 536.21

We examine original solutions for two-dimensional problems of nonsteady heat conduction in the case of an orthotropic half-space with discontinuous mixed boundary conditions; these are used to develop new methods of controlling thermophysical properties in a nondestructive manner.

One of the most important problems confronting experimental thermophysics is the elevation of the accuracy and productivity of thermophysical measurements, convenience in their practical realization from a standpoint both of methodology and engineering. In comparison with many methods of determining thermophysical characteristics (TPC) in materials, greatest preference is presently given to nonsteady methods and means which ensure complex thermophysical measurements in various materials, without destruction of their natural structure and integrity (the so-called methods of means of nondestructive TPC control) [3-8]. The decisions of recent international and all-union thermophysical conferences on the properties of materials have recently stressed particular attention on the need for promising developments in the attainment of nondestructive control of TPC.

The proposed methods of nondestructive control (TPC) can be incorporated under the concept of precision methods, since the hypothetical boundary conditions are rather precise and can easily be achieved in actual practice.

In the area of measurement techniques involving nonelectrical quantities, the concept of a precision instrument (device) is constantly accompanied by the concept of the precision method. The metrological aspect of the need to unify these concepts is felt most urgently in the techniques of thermophysical experimentation. The methodological error in the measurement of thermophysical properties is determined primarily by the extent of divergence between theoretically postulated boundary conditions and those actually encountered in practice.

If the hypothetical boundary conditions are satisfied to a sufficient degree of accuracy during the course of a thermophysical experiment, we can render judgement as to the real accuracy of these thermophysical measurements, since only those that are associated with instrumental error in the determination of the TPC will be dealt with in our analysis of the errors. However, methods of reducing these errors are widely known and involve the utilization of perfected (precise) measurement capabilities to determine individual quantities included in the appropriate theoretical formulas for the determination of TPC. In the actual practice of thermophysical measurements, two forms of boundary conditions are most easily realized: constancy of temperature at the surface of the test body; constancy of the heat flow, provided that the latter is generated by means of a low-inertia electric heater

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 57, No. 6, pp. 999-1005, December, 1989. Original article submitted May 30, 1988.

Let us examine the following two-dimensional problem of nonsteady heat conduction: semibounded (orthotropic) body with a constant initial temperature T_0 = const is heated through a circular region ($0 \le r < R$, $z = 0$) by a constant flow of heat with density q_0. Beyond the limits of this circular region ($r > R$, $z = 0$), over the entire range in which exchange of heat takes place, a constant temperature is maintained, which is equal to the initial temperature T_0. We are called to find a solution for the following system of differential equations for the functions $\Phi(r, z, \tau) = \theta_1 = T_1(r, z, \tau) - T_0 (0 \le r < R, z > 0, \tau > 0)$ and $\theta_2(r, z, \tau) = \theta_2 = T_2(r, z, \tau) - T_0 (r > R, z > 0, \tau > 0)$:

$$\frac{\partial^2\Theta_1}{\partial r^2} + \frac{1}{r}\frac{\partial\Theta_1}{\partial r} + \frac{a_z}{a_r}\frac{\partial^2\Theta_1}{\partial z^2} = \frac{1}{a_r}\frac{\partial\Theta_1}{\partial\tau}, \quad 0 \leqslant r < R; \tag{1}$$

$$\frac{\partial^2\Theta_2}{\partial r^2} + \frac{1}{r}\frac{\partial\Theta_2}{\partial r} + \frac{a_z}{a_r}\frac{\partial^2\Theta_2}{\partial z^2} = \frac{1}{a_r}\frac{\partial\Theta_2}{\partial\tau}, \quad R < r < \infty, \tag{2}$$

under the following boundary conditions:

$$\Theta_1(r, z, 0) = \Theta_2(r, z, 0) = 0, \tag{3}$$

$$-\frac{\partial\Theta_1(r, 0, \tau)}{\partial z} = \frac{q_0}{\lambda_z}, \quad 0 \leqslant r < R, \quad z = 0, \quad \tau > 0, \tag{4}$$

$$\Theta_2(r, 0, \tau) = 0, \quad r > R, \quad z = 0, \quad \tau > 0, \tag{5}$$

$$\frac{\partial\Theta_1(0, z, \tau)}{\partial r} = 0, \quad r = 0, \quad z > 0, \quad \tau > 0, \tag{6}$$

$$\frac{\partial\Theta_1(r, \infty, \tau)}{\partial z} = \frac{\partial\Theta_2(r, \infty, \tau)}{\partial z} = \frac{\partial\Theta_2(\infty, z, \tau)}{\partial r} = 0, \tag{7}$$

$$\Theta_1(R, z, \tau) = \Theta_2(R, z, \tau), \quad z > 0, \quad \tau > 0, \tag{8}$$

$$\frac{\partial\Theta_1(R, z, \tau)}{\partial r} = \frac{\partial\Theta_2(R, z, \tau)}{\partial r}, \quad z > 0, \quad \tau > 0. \tag{9}$$

In our subsequent studies we will be interested in finding a solution for $\theta_1(r, z, \tau)$ in the first region ($0 \le r < R$, $z \ge 0$, $\tau > 0$), which in the space of L images, with consideration of boundary conditions (3)-(9), can be written in the form

$$\overline{\Theta}_1(r, z, s) = \frac{2q_0\sqrt{a_z}}{\pi\lambda_z s}\int\limits_0^\infty \frac{\exp\left(-\frac{z}{\sqrt{a_z}}\sqrt{s + \frac{a_r x^2}{R^2}}\right)}{\sqrt{s + \frac{a_r x^2}{R^2}}} \times J_0\left(\frac{r}{R}x\right)\frac{\sin x - x\cos x}{x}\,dx, \quad 0 \leqslant r < R. \tag{10}$$

When $a_r = a_z = a$, $\lambda_r = \lambda_z = \lambda$, from (10) we have the corresponding solution for an isotropic (semibounded in the thermal sense) body while as $R \to \infty$ from (10) we have the familiar [1] one-dimensional solution for the semibounded isotropic medium heated by the constant heat flow of density q_0.

Applying the inverse Laplace transform to (10), we obtain

$$\Theta_1(r, z, \tau) = \frac{q_0 R}{\pi\lambda_z\sqrt{K_a}}\int\limits_0^\infty J_0\left(\frac{r}{R}x\right)\frac{\sin x - x\cos x}{x^2} \times \left\{\exp\left(-\frac{z}{R}\sqrt{K_a}\,x\right)\operatorname{erfc}\left(\frac{z}{2\sqrt{a_z\tau}} - \frac{x}{R}\sqrt{a_r\tau}\right) - \exp\left(\frac{z}{R}\sqrt{K_a}\,x\right)\operatorname{erfc}\left(\frac{z}{2\sqrt{a_z\tau}} + \frac{x}{R}\sqrt{a_r\tau}\right)\right\}dx. \tag{11}$$

At the axis $r = 0$ ($z, \tau \ge 0$) solution (11) assumes the form

$$\Theta_1(0, z, \tau) = \frac{2q_0\sqrt{\tau}}{b_z\sqrt{\pi}}\left\{\exp\left(-\frac{z^2}{4a_z\tau}\right)\operatorname{erf}\left(\frac{R}{2\sqrt{a_r\tau}}\right) - \frac{z}{\sqrt{\pi a_z\tau}}\left[\operatorname{arctg}\left(\frac{R}{z\sqrt{K_a}}\right) - \frac{\sqrt{\pi}}{2}\int\limits_0^{\frac{z}{2\sqrt{a_z\tau}}}\operatorname{erf}\left(\frac{Rx}{z\sqrt{K_a}}\right)\exp(-x^2)\,dx\right]\right\}. \tag{12}$$

The excess temperature at the center of the heating spot (r = z = 0) at the surface of this (orthotropic) body is determined from the following expression:

$$\Theta_1(0,\ 0,\ \tau) = \frac{2q_0\sqrt{\tau}}{b_z\sqrt{\pi}}\,\mathrm{erf}\left(\frac{R}{2\sqrt{a_r\tau}}\right). \tag{13}$$

In the steady thermal regime ($\tau \to \infty$) solution (11) exhibits an analytical extension into the second region r > R. Thus, for any point r and z ($0 \le r < \infty$, $0 \le z < \infty$) as $\tau \to \infty$, we have

$$\Theta_i(r,\ z,\ \infty) = \frac{2q_0R}{\pi\lambda_z\sqrt{K_a}}\int_0^\infty \exp\left(-\frac{z}{R}\sqrt{K_a}\,x\right)J_0\left(\frac{r}{R}\,x\right)\ \frac{\sin x - x\cos x}{x^2}\,dx,\quad i = 1,\ 2. \tag{14}$$

If $K_a = 1$, then from (14) we confined the corresponding steady-state solution for the isotropic half-space heated through the circular region ($0 \le r < R$, $z = 0$) by a constant flow of heat, with the temperature T_0 kept constant outside of the circle ($r > R$, $z = 0$) at the boundary of this body [2].

When z = 0, from (14) we have a value (distribution) for the steady temperature at the surface of the orthotropic half-space in the area of the circular heating spot ($0 \le r \le \infty$):

$$\Theta_i(r,\ 0,\ \infty) = \begin{cases} \dfrac{2q_0R}{\pi\lambda_z\sqrt{K_a}}\sqrt{1-\dfrac{r^2}{R^2}}, & r \leqslant R; \\ 0, & r \geqslant R. \end{cases} \tag{15}$$

For the steady-state value of the excess temperature (14) at the axis r = 0 ($z \ge 0$) we have

$$\Theta_1(0,\ z,\ \infty) = \frac{2q_0R}{\pi\lambda_z\sqrt{K_a}}\left\{1-\frac{z}{R}\sqrt{K_a}\,\mathrm{arctg}\left(\frac{R}{z\sqrt{K_a}}\right)\right\}. \tag{16}$$

According to (15) and (16), the steady-state value of the excess temperature at the central point (r = z = 0) of the circulating spot assumes the following simple form:

$$\Theta_1(0,\ 0,\ \infty) = \frac{2q_0R}{\pi\lambda_z\sqrt{K_a}}. \tag{17}$$

Using (14), it is not difficult to find the ratio between the heat-flux density $\lambda_z \times [\partial\theta_i(r,\ 0,\ \infty)/\partial z] = q^*(r)$ at any point on the boundary surface z = 0, $r \ge 0$ to the given heat-flux density q_0 = const in the circle region ($0 \le r < R$, $z = 0$):

$$\left.\frac{q^*(r)}{q_0}\right|_{z=0} = \begin{cases} -1, & r < R; \\ \dfrac{2}{\pi}\left[\dfrac{1}{\sqrt{\dfrac{r^2}{R^2}-1}} - \arcsin\left(\dfrac{R}{r}\right)\right], & r > R. \end{cases} \tag{18}$$

Relationship (18) is represented in Fig. 1. On the basis of this representation, which characterizes the behavior of the discontinuous (generated) heat-flux function, where the flow of heat is directed normally to the boundary of the surface (z = 0) of the body being examined. From the standpoint of actual practice we can draw the following important conclusion with regard to the extent to which the flow of heat is reduced in intensity in the cooling region (r > R, z = 0), i.e., in the region at which the temperature T_0 is kept constant (at the surface of the body, i.e., z = 0, r > R). These conclusions may find application in various branches of science and engineering; among these we might include an optimum choice for the dimensions of bodies being studied under the experimental techniques of thermophysics, proceeding from a given measurement accuracy with regard to thermophysical characteristics.

On the basis of the above analytical material, we propose original calculation formulas for the determination of the thermophysical properties of orthotropic materials (without destroying their integrity), provided that the theoretically postulated boundary conditions (3)-(9) are realized as part of the technique of the thermophysical experiment:

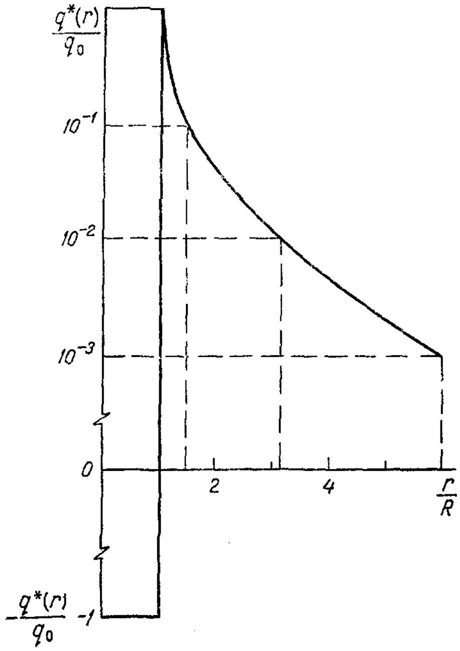

Fig. 1. Distribution of heat-flux density, normal to the heating surface (z = 0) as a ratio of the given q_0 in the circular region ($0 \le r < R$, z = 0), as a function of the relative radius r/R in the steady-state thermal regime [relationship (18)].

1. Calculation of the Coefficient of Thermal Activity $b_z = \lambda_z / \sqrt{a_z}$. From (13) as $\tau \to 0$ ($a_r\tau/R^2 < 0.1$) we have

$$b_z = \frac{2q_0\sqrt{\tau}}{\Theta_1(0,\ 0,\ \tau)\sqrt{\pi}}. \tag{19}$$

2. Determination of the Coefficient of Thermal Diffusivity a_r.

2.1 With known values for b_z, we determine the coefficient a_r from the equation

$$Y = \operatorname{erf}(X), \tag{20}$$

where

$$Y = \frac{\Theta_1(0,\ 0,\ \tau)\, b_z \sqrt{\pi}}{2q_0\sqrt{\tau}}, \tag{21}$$

$$X = \frac{R}{2\sqrt{a_r\tau}}. \tag{22}$$

From the experimentally derived values of Y, using (20), we find the value of the argument X. Then

$$a_r = \frac{R^2}{4\tau X^2}. \tag{23}$$

2.2. With the values of b_z unknown, the coefficient a_r is determined from the equation

$$N = \frac{\Theta_1(0,\ 0,\ \tau_1)}{\Theta_1(0,\ 0,\ k\tau_1)} = \frac{\operatorname{erf}\left(\frac{1}{2\sqrt{\mathrm{Fo}_1}}\right)}{\operatorname{erf}\left(\frac{1}{2\sqrt{k\,\mathrm{Fo}_1}}\right)}\,\frac{1}{\sqrt{k}}, \tag{24}$$

where $k = \tau_2/\tau_1$ represents the number of measurements in the ratio (24) specified in advance.

From the experimentally derived values of N, using (24), we find the value of the Fo_1 number. In this case

$$a_r = \frac{R^2}{\tau_1}\mathrm{Fo}_1. \tag{25}$$

3. Determination of the Complex $\lambda_z\sqrt{K_a} = \lambda_r/\sqrt{K_a} = \lambda_r/\sqrt{K_\lambda} = c\gamma\sqrt{a_z a_r} = \sqrt{\lambda_z\lambda_r} = b_z\sqrt{a_r} = b_r\sqrt{a_z}$.

From (15) we have

$$\lambda_z \sqrt{K_a} = \frac{2q_0 R}{\pi\Theta_1(r,\ 0,\ \infty)} \sqrt{1 - \frac{r^2}{R^2}}, \tag{26}$$

or when r = 0

$$\lambda_z \sqrt{K_a} = \frac{2q_0 R}{\pi\Theta_1(0,\ 0,\ \infty)}. \tag{27}$$

4. Determination of the Parameter $K_a = a_r/a_z = K_\lambda = \lambda_r/\lambda_z$. The parameter K_a, using (16), is found from the transcendental equation

$$\operatorname{arctg} X = AX, \tag{28}$$

where

$$A = 1 - \frac{\Theta_1(0,\ z,\ \infty)}{\Theta_1(0,\ 0,\ \infty)}, \quad z > 0, \tag{29}$$

$$X = \frac{R}{z\sqrt{K_a}}. \tag{30}$$

Equation (28) always has roots, since A < 1. Having determined x, K_a we find from the following formula that

$$\sqrt{K_a} = \frac{R/z}{X}, \tag{31}$$

or

$$K_a = \left(\frac{R/z}{X}\right)^2. \tag{32}$$

The thermal conductivity λ_z for known K_a is found from (27), while the thermal diffusivity a_z is found from the formula

$$a_z = \frac{a_r}{K_a}. \tag{33}$$

We calculate the volumetric heat capacity $c\gamma$ by means of the following formula

$$c\gamma = \frac{1}{\sqrt{a_r a_z}} \frac{2q_0 R}{\pi\Theta_1(0,\ 0,\ \infty)}. \tag{34}$$

If the body is isotropic ($K_a = 1$), then the complex determination of the TPC (proceeding from the solution of this problem) can be obtained without distroying the integrity of the material being studied, provided that we use the exact formulas (24) and (27), since in this case $b = \lambda/\sqrt{a}$, $a\, c\gamma = \lambda$.

The complex determination of the TPC of an orthotropic body (without destroying the integrity of the test material) can be achieved in actual practice by using a combination experimental method. Essentially this involves the following: if the test material exhibits a finite dimension h in the direction of the z axis and a constant temperature equal to T_0 is maintained at the z = h surface, and if in the place of condition (5) we establish the condition of ideal thermal insulation, then on the basis of the solution of the corresponding heat-conduction problem, the sort parameter K_a can be determined in the steady state from the following equation:

$$N_1 = \frac{1 - \dfrac{4}{\pi\bar{h}\sqrt{K_a}} \sum_{n=0}^{\infty} \dfrac{1}{2n+1} K_1\left[\dfrac{\pi(2n+1)}{2\bar{h}\sqrt{K_a}}\right] I_0\left[\dfrac{\pi(2n+1)}{2\bar{h}\sqrt{K_a}}\bar{r}\right]}{1 - \dfrac{4}{\pi\bar{h}\sqrt{K_a}} \sum_{n=0}^{\infty} \dfrac{1}{2n+1} K_1\left[\dfrac{\pi(2n+1)}{2\bar{h}\sqrt{K_a}}\right]}, \tag{35}$$

where $N_1 = [T(r,\ 0,\ \infty) - T_0]/[T(0,\ 0,\ \infty) - T_0]$ is the experimentally established ratio of excess temperatures at the points r = r, z = 0 and r = z = 0 in the steady regime; $\bar{r} = r/R$; $\bar{h} = h/R$.

Additionally, in the steady-state, for a plane orthotropic layer of thickness h, heated by a circular source of constant-power heat, we have the theoretical formula for the determination of the coefficient of thermal conductivity λ_z:

$$\lambda_z = \frac{q_0 h}{T(0,\ 0,\ \infty) - T_0}\left\{1 - \frac{4}{\pi \bar{h} \sqrt{K_a}} \sum_{n=0}^{\infty} \frac{K_1\left[\frac{\pi(2n+1)}{2\bar{h}\sqrt{K_a}}\right]}{2n+1}\right\}. \quad (36)$$

Thus, on the basis of the derived solution and the analytical studies which we carried out into this two-dimensional problem of nonsteady heat conduction in the presence of mixed discontinuous boundary conditions of the first and second kind, we propose new calculation formulas for the complex determination of the TPC of isotropic and orthotropic materials, using as our basis the physical model of the half-space and an unbounded plate. The determination of all of the thermophysical parameters for the objects being investigated is possible without destruction of their integrity, i.e., all of the necessary calculations of the TPC parameters can be carried out in accordance with the results from corresponding temperature measurements exclusively at the boundary surface (z = 0) of the media being considered. The proposed methods of nondestructive TPC control might serve as an analytical basis (mathematical foundation) for the design of contemporary microprocessor measuring facilities (systems) in the construction of thermophysical instrumentation.

NOTATION

$\theta_i(r, z, \tau) = T_i(r, z, \tau) - T_0$, excess temperature of the material being examined, in the i-th region; T_0, initial temperature; R, radius of circular source; $\bar{h} = h/R$, relative height (thickness) of unbounded plates; r, z, τ, cylindrical coordinates and time; s, Laplace transform parameter; a_r, a_z, λ_r, λ_z, b_r, b_z, respectively, the thermal diffusivity, thermal conductivity, and thermal activity in the direction of the coordinates r and z; $b_i = \lambda_i/\sqrt{a_i}$; q_0, heat-flux density (W/m²); $J_0(x)$, $K_1(x)$, respectively, Bessel function of zeroth order for the real argument and the MacDonald first-order function; erf(x), probability integral; $q^*(r)$, heat-flux density, normal to the boundary heating surface, as a function of the point r (z = 0); N, Y, A, experimentally measured parameters; $k = \tau_2/\tau_1$, measurement repetition multiple; $K_a = a_r/a_z = K_\lambda = \lambda_r/\lambda_z$, relationships between the thermophysical properties in an orthotropic body.

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